22.03.2006

0.1 65. Hausaufgabe

0.1.1 Geometrie-Buch Seite 40, Aufgabe 7

Berechne und vereinfache.

a) $\begin{vmatrix} 1 & 1 & 0 \\ 1 & 1+a & 0 \\ 1 & 1 & 1+b \end{vmatrix} = (1+b)\begin{vmatrix} 1 & 1 \\ 1 & 1+a \end{vmatrix} = (1+b)(1+a-1) = a+ab;$

b) $\begin{vmatrix} 1 & a & -b \\ -a & 1 & c \\ b & -c & 1 \end{vmatrix} = \begin{vmatrix} 1 & c \\ -c & 1 \end{vmatrix} + a\begin{vmatrix} a & -b \\ -c & 1 \end{vmatrix} + b\begin{vmatrix} a & -b \\ 1 & c \end{vmatrix} =$
$1 + c^2 + a^2 - abc + abc + b^2 = 1 + a^2 + b^2 + c^2;$

c) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = \begin{vmatrix} b & c \\ b^2 & c^2 \end{vmatrix} - \begin{vmatrix} a & c \\ a^2 & c^2 \end{vmatrix} + \begin{vmatrix} a & b \\ a^2 & b^2 \end{vmatrix} =$
$bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b = a^2(c-b) + b^2(a-c) + c^2(b-a);$

d) $\begin{vmatrix} a & b & a+b \\ b & a+b & a \\ a+b & a & b \end{vmatrix} = a\begin{vmatrix} a+b & a \\ a & b \end{vmatrix} - b\begin{vmatrix} b & a+b \\ a & b \end{vmatrix} + (a+b)\begin{vmatrix} b & a+b \\ a+b & a \end{vmatrix} =$
$a(ab+b^2-a^2) - b(b^2-a^2-ab) + (a+b)(ab-a^2-2ab-b^2) = -2a^3 - 2b^3;$

e) $\begin{vmatrix} \sin\alpha & \cos\alpha\tan\beta & \cos\alpha \\ -\cos\alpha & \sin\alpha\tan\beta & \sin\alpha \\ 0 & -1 & \tan\beta \end{vmatrix} = \begin{vmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{vmatrix} + \tan\beta\begin{vmatrix} \sin\alpha & \cos\alpha\tan\beta \\ -\cos\alpha & \sin\alpha\tan\beta \end{vmatrix} =$
$\sin^2\alpha + \cos^2\alpha + \tan^2\beta\left(\sin^2\alpha + \cos^2\alpha\right) = 1 + \tan^2\beta;$

0.1.2 Geometrie-Buch Seite 40, Aufgabe 9

Löse die Gleichungssysteme mit der Cramer-Regel.

a) $\begin{array}{rcrcrcl} 2x_1 & + & x_2 & + & 5x_3 & = & 1; \\ 2x_1 & + & 4x_2 & + & x_3 & = & 1; \\ x_1 & + & x_2 & + & 2x_3 & = & 1; \end{array}$

$$D = \begin{vmatrix} 2 & 1 & 5 \\ 2 & 4 & 1 \\ 1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 5 \\ 4 & 1 \end{vmatrix} - \begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 2 & 4 \end{vmatrix} = -19 + 8 + 12 = 1 \neq 0;$$

$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} 1 & 1 & 5 \\ 1 & 4 & 1 \\ 1 & 1 & 2 \end{vmatrix}}{D} = \begin{vmatrix} 4 & 1 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 5 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 5 \\ 4 & 1 \end{vmatrix} = 7 + 3 - 19 = -9;$$

$$x_2 = \frac{D_2}{D} = \frac{\begin{vmatrix} 2 & 1 & 5 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix}}{D} = -\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix} - \begin{vmatrix} 2 & 5 \\ 2 & 1 \end{vmatrix} = -3 - 1 + 8 = 4;$$

$$x_3 = \frac{D_3}{D} = \frac{\begin{vmatrix} 2 & 1 & 1 \\ 2 & 4 & 1 \\ 1 & 1 & 1 \end{vmatrix}}{D} = \begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix} - \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 2 & 4 \end{vmatrix} = -2 - 1 + 6 = 3;$$

$(x_1, x_2, x_3) = (-9, 4, 3);$

b)
$$\begin{array}{rcrcrcr} 3x_1 & + & 5x_2 & + & 3x_3 & = & 1; \\ 2x_1 & + & -x_2 & + & -x_3 & = & -2; \\ x_1 & + & 3x_2 & + & 2x_3 & = & -1; \end{array}$$

$$D = \begin{vmatrix} 1 & 5 & 3 \\ 2 & -1 & -1 \\ 1 & 3 & 2 \end{vmatrix} = -2\begin{vmatrix} 5 & 3 \\ 3 & 2 \end{vmatrix} - \begin{vmatrix} 3 & 3 \\ 1 & 2 \end{vmatrix} + 2\begin{vmatrix} 3 & 5 \\ 1 & 3 \end{vmatrix} = -2 - 3 + 4 = -1 \neq 0;$$

$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} 1 & 5 & 3 \\ -2 & -1 & -1 \\ -1 & 3 & 2 \end{vmatrix}}{D} = -\left(2\begin{vmatrix} 5 & 3 \\ 3 & 2 \end{vmatrix} - \begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 5 \\ -1 & 3 \end{vmatrix}\right) = -(2 - 5 + 8) = -5;$$

$$x_2 = \frac{D_2}{D} = \frac{\begin{vmatrix} 3 & 1 & 3 \\ 2 & -1 & -1 \\ 1 & -1 & 2 \end{vmatrix}}{D} = -\left(\begin{vmatrix} 1 & 3 \\ -2 & -1 \end{vmatrix} + \begin{vmatrix} 3 & 3 \\ 2 & -1 \end{vmatrix} + 2\begin{vmatrix} 3 & 1 \\ 2 & -2 \end{vmatrix}\right) = -(5 - 9 - 16) = 20;$$

$$x_3 = \frac{D_3}{D} = \frac{\begin{vmatrix} 3 & 5 & 1 \\ 2 & -1 & -2 \\ 1 & 3 & -1 \end{vmatrix}}{D} = -\left(\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} + 2\begin{vmatrix} 3 & 5 \\ 1 & 3 \end{vmatrix} - \begin{vmatrix} 3 & 5 \\ 2 & -1 \end{vmatrix}\right) = -(7 + 8 + 13) = -28;$$

$(x_1, x_2, x_3) = (-5, 20, -28);$