

# Plasma vs Drude modelling of the Casimir force: beyond the proximity force approximation

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aspect ratio:  $\frac{R}{L} \leftarrow$  sphere radius  
 $\frac{R}{L} \leftarrow$  separation

## experiments

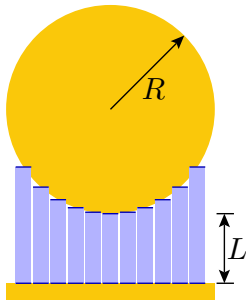
- ▶ large force  $\rightarrow$  large radius of sphere
- ▶ aspect ratio  $R/L \sim 10^3 - 10^4$

## theory

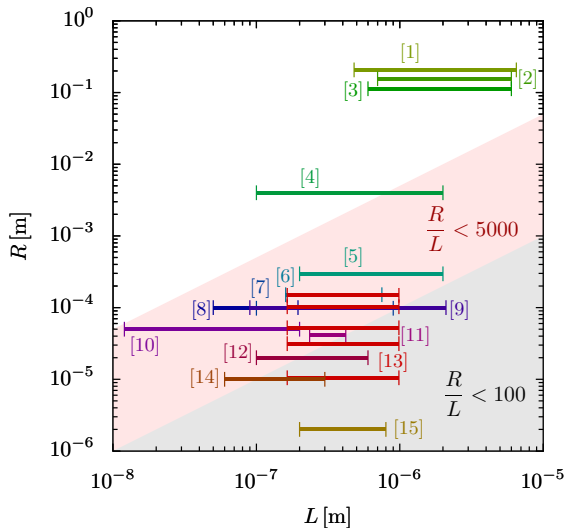
- ▶ scattering approach for *small* aspect ratios  
 but:  $\ell_{\text{dim}} \sim R/L$ , ill-conditioned matrices
- ▶ Proximity Force Approximation (PFA)

$$F_{\text{PFA}} = 2\pi R \mathcal{F}_{\text{PP}}, \quad \mathcal{F}_{\text{PP}}: \text{energy for two planes}$$

for *large* aspect ratios (Q62.56, Spreng, Thu 16:15–18:15)



*Accuracy of PFA? Corrections?*



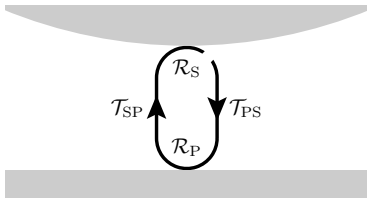
- [1] Masuda, Sasaki (2009)
- [2] Sushkov *et al.* (2011)
- [3] Lamoreaux (1997)
- [4] Garcia-Sanchez *et al.* (2012)
- [5] Decca *et al.* (2003)
- [6] Decca *et al.* (2007)
- [7] Mohideen, Roy (1998)
- [8] Man *et al.* (2009)
- [9] Chan *et al.* (2001)
- [10] van Zwol *et al.* (2008)
- [11] Chang *et al.* (2012)
- [12] Jourdan *et al.* (2009)
- [13] Krause *et al.* (2007)
- [14] Torricelli *et al.* (2011)
- [15] Ether *et al.* (2015)

Free energy

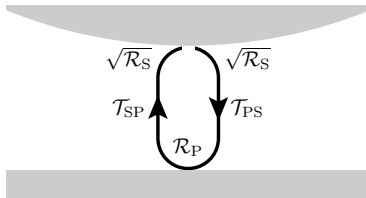
$$\mathcal{F} = \frac{k_B T}{2} \sum_{n=-\infty}^{\infty} \log \det [\mathbb{I} - \mathcal{M}(|\xi_n|)], \quad \xi_n = \frac{2\pi n k_B T}{\hbar}$$

Roundtrip operator

$$\mathcal{M} = \mathcal{R}_S \mathcal{T}_{SP} \mathcal{R}_P \mathcal{T}_{PS}$$



$$\widehat{\mathcal{M}} = \sqrt{\mathcal{R}_S} \mathcal{T}_{SP} \mathcal{R}_P \mathcal{T}_{PS} \sqrt{\mathcal{R}_S}$$

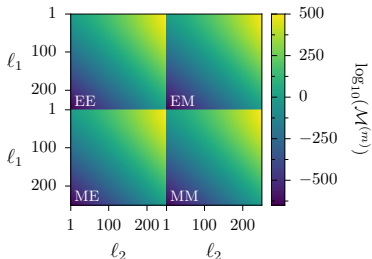


## Free energy

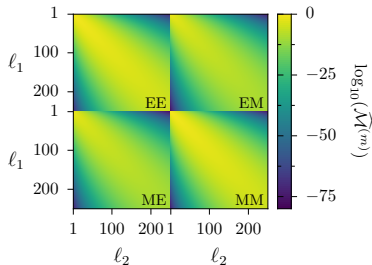
$$\mathcal{F} = \frac{k_B T}{2} \sum_{n=-\infty}^{\infty} \log \det [\mathbb{I} - \mathcal{M}(|\xi_n|)], \quad \xi_n = \frac{2\pi n k_B T}{\hbar}$$

## Roundtrip operator

$$\mathcal{M} = \mathcal{R}_S \mathcal{T}_{SP} \mathcal{R}_P \mathcal{T}_{PS}$$



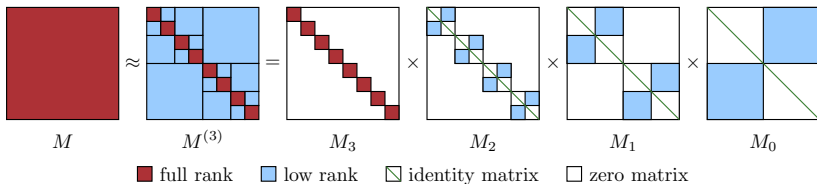
$$\widehat{\mathcal{M}} = \sqrt{\mathcal{R}_S} \mathcal{T}_{SP} \mathcal{R}_P \mathcal{T}_{PS} \sqrt{\mathcal{R}_S}$$



- ▶ sub-divide matrix into hierarchy of rectangular blocks
- ▶ approximate blocks by low rank approximation
- ▶ *fast* computation of determinant in

$$\mathcal{O}(N \log^2 N)$$

of an  $N \times N$  matrix



## Drude model

- ▶ takes dissipation into account
- ▶ finite conductivity
- ▶ TE mode not reflected for  $\xi = 0$

## Plasma model

- ▶ no dissipation
- ▶ no finite conductivity
- ▶ TE mode partially reflected for  $\xi = 0$

## Plasma-Drude controversy

- ▶ **How to treat  $\xi = 0$ ?** Plasma or Drude?
- ▶ differences in the high-temperature limit
- ▶ some experiments seem to agree with the Plasma model

*Maybe the discrepancy comes from corrections to PFA?*

## Experimental Investigation of the Casimir Force beyond the Proximity-Force Approximation

D. E. Krause,<sup>1,2</sup> R. S. Decca,<sup>3</sup> D. López,<sup>4</sup> and E. Fischbach<sup>2</sup>

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<sup>4</sup>*Bell Laboratories, Lucent Technologies, Murray Hill, New Jersey 07974, USA*

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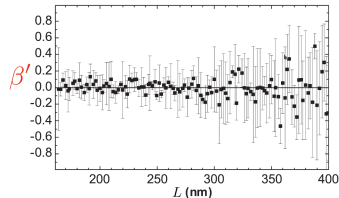
The analysis of all Casimir force experiments using a sphere-plate geometry requires the use of the proximity-force approximation (PFA) to relate the Casimir force between a sphere and a flat plate to the Casimir energy between two parallel plates. Because it has been difficult to assess the PFA's range of applicability theoretically, we have conducted an experimental search for corrections to the PFA by measuring the Casimir force and force gradient between a gold-coated plate and five gold-coated spheres with different radii using a microelectromechanical torsion oscillator. For separations  $z < 300$  nm, we find that the magnitude of the fractional deviation from the PFA in the force gradient measurement is, at the 95% confidence level, less than  $0.4z/R$ , where  $R$  is the radius of the sphere.

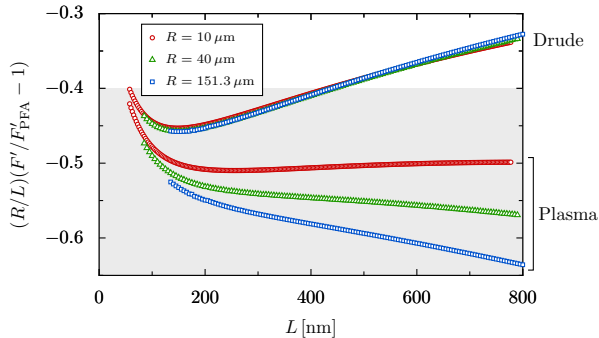
DOI: 10.1103/PhysRevLett.98.050403

PACS numbers: 12.20.Fv, 03.70.+k, 42.50.Lc

- ▶  $T = 295$  K, gold surfaces
- ▶ corrections,  $|\beta'| \leq 0.4$

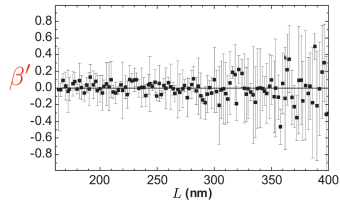
$$\frac{F'}{F'_{\text{PFA}}} - 1 = \beta'(L) \frac{L}{R} + \dots$$

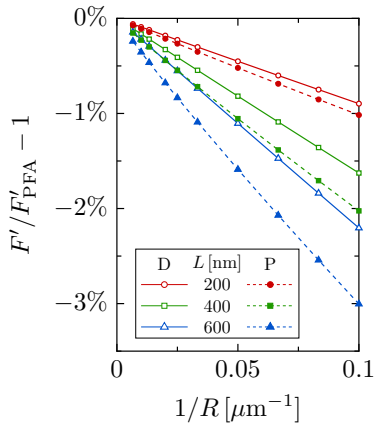
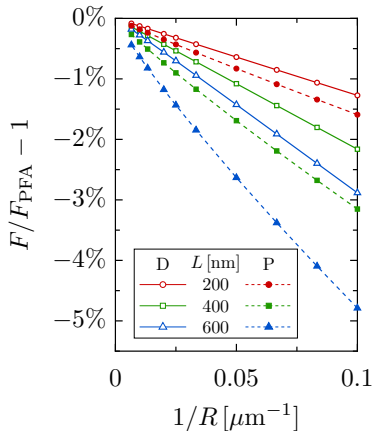




- $T = 295$  K, gold surfaces
- corrections,  $|\beta'| \leq 0.4$

$$\frac{F'}{F'_{\text{PFA}}} - 1 = \beta'(L) \frac{L}{R} + \dots$$





## Significant numerical advances

- ▶ symmetrization of round-trip operator
  - ▶ use of hierarchical matrices
- ⇒ numerics now capable to describe experiments

## corrections to PFA

- ▶ experimental bound for corrections to PFA violated for Drude and plasma model
- ▶ corrections do not resolve plasma-Drude controversy
- ▶ but: bound stronger violated by plasma model
- ▶ corrections for force gradient smaller than for force