2013 F / Geladenes Teilchen

$\vec{E} = \vec{E_y}, \quad \vec{B} = B\vec{e_z}$

a) Lorentzkraft: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = q\left[\begin{pmatrix}0\\E\\0\end{pmatrix} + B\begin{pmatrix}v_y\\-v_x\\0\end{pmatrix}\right]$

Bewegungsgl: $\vec{F} = m\vec{a} = m\dot{\vec{v}}, \quad \begin{pmatrix}\dot{v}_x\\\dot{v}_y\\\dot{v}_z\end{pmatrix} = \frac{q}{m}\begin{pmatrix}Bv_y\\E - Bv_x\\0\end{pmatrix}$ $\Rightarrow v_z(t) = v_z(0) = 0$

$\dot{v}_x = \frac{qB}{m} v_y \quad \Leftrightarrow \quad v_y = \frac{m}{qB}\dot{v}_x$

$\dot{v}_y = \frac{m}{qB}\ddot{v}_x = \frac{qE}{m} - \frac{qB}{m}v_x$

$\ddot{v}_x + \omega^2 v_x - \frac{q^2 EB}{m^2} = 0 \quad \text{mit } \omega = \frac{qB}{m}$

homogene Lsg: $v_x(t) = A_1 \sin\omega t + A_2 \cos\omega t$

spezielle Lsg: $v_x(t) = \frac{q^2 EB}{m^2\omega^2} = \frac{E}{B}$

komplette Lsg = homogene Lsg + spezielle Lsg

$v_x(t) = A_1 \sin\omega t + A_2 \cos\omega t + \frac{E}{B}$

$v_y(t) = \underbrace{\frac{m}{qB}}_{=\frac{1}{\omega}} \omega(A_1\cos\omega t - A_2 \sin\omega t) = A_1 \cos\omega t - A_2 \sin\omega t$

Mit AB: $v_x(0) = A_2 + \frac{E}{B} \overset{!}{=} 0 \quad \Rightarrow A_2 = -\frac{E}{B}$

$v_y(0) = A_1 \overset{!}{=} 0 \quad \Rightarrow A_1 = 0$

$\vec{v} = \frac{E}{B}\begin{pmatrix}1 - \cos\omega t\\+\sin\omega t\\0\end{pmatrix}$

$\vec{x} = \int_0^t dt'\, v_x(t') = \frac{E}{B}t - \frac{E}{B\omega}\sin\omega t$

$\vec{y} = \int_0^t dt'\, v_y(t') = -\frac{E}{B\omega}\cos\omega t'\Big|_0^t = \frac{E}{B\omega}(1 - \cos\omega t)$

$\vec{z} = 0$

b) $m\dot{\vec{v}} = q(\vec{E} + \vec{v} \times \vec{B})$

$\Rightarrow m\vec{v}\cdot\dot{\vec{v}} = q\vec{E}\cdot\vec{v} + q\vec{v}\cdot(\vec{v}\times\vec{B})$

$0 = m\vec{v}\cdot\dot{\vec{v}} - q\vec{E}\cdot\vec{v} = \frac{d}{dt}\frac{1}{2}m\vec{v}^2 - q\dot{\vec{r}}\cdot\vec{E} = \frac{d}{dt}\underbrace{\left(\frac{1}{2}m\vec{v}^2 - q\vec{r}\cdot\vec{E}\right)}_{=\mathcal{E}} = \frac{d}{dt}\mathcal{E}$

Energie: $\mathcal{E} = \frac{1}{2}mv^2 - qyE$

Einsetzen: $\mathcal{E} = \frac{1}{2}m\frac{E^2}{B^2}(1 - 2\cos\omega t + \cos^2\omega t + \sin^2\omega t) - \frac{qE^2}{B\omega}(\cos\omega t - 1) = m\frac{E^2}{B^2}(1 - \cos\omega t) - m\frac{E^2}{B^2}(1 - \cos\omega t) = 0 = \text{const}$

siehe b)

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{E}{B\omega} \begin{pmatrix} \omega t - \sin\omega t \\ 1 - \cos\omega t \end{pmatrix}$$